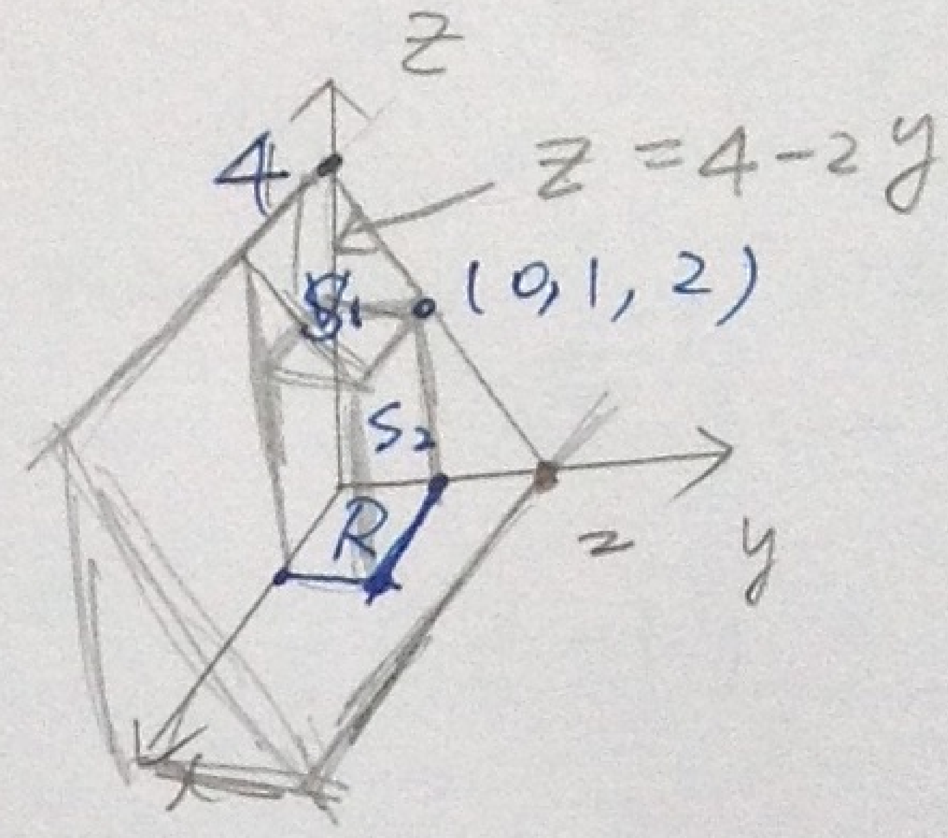
7–9. Evaluate the double integral by first identifying it as the volume of a solid.

#9. $\iint_R (4-2y)\,dA$, $R=[0,1]\times[0,1]$

sol: $z=4-2y$ a plane



以 $z=4-2y$ 為屋頂

以 R 為地基之 solid

$= S_1 \cup S_2$

上面 (長方体之一半)　下面(長方体)

$V_1 = 1\times1\times2\times\frac{1}{2} = 1$, $V_2 = 1\times1\times2 = 2$

$\therefore \iint_R (4-2y)\,dA = V_1+V_2 = \underline{3}$ ※

11–20. Calculate the iterated integral.

#11. $\int_1^3\int_0^1 (1+4xy)\,dx\,dy = \int_1^3 (x+2x^2y)\Big|_{x=0}^{x=1} dy$

$= \int_1^3 (1+2y)\,dy = y+y^2\Big|_{y=1}^{y=3} = \underline{10}$ ※

#13 $\int_0^2\int_0^{\frac{\pi}{2}} x\sin y\,dy\,dx = \int_0^2 (-x\cos y)\Big|_{y=0}^{y=\frac{\pi}{2}} dx$

$= \int_0^2 x\,dx = \underline{2}$ ※

〈或法二〉

$\int_0^2 x\,dx \cdot \int_0^{\frac{\pi}{2}} \sin y\,dy = 2\times1 = 2$

#15 $\int_0^2 \underbrace{\int_0^1 (2x+y)^8\,dx}_{2\cdot\frac{1}{2}}\,dy = \int_0^2 \left(\frac{1}{9}\times\frac{1}{2}\times(2x+y)^9\Big|_{x=0}^{x=1}\right) dy$

$= \frac{1}{18}\int_0^2 [(y+2)^9 - y^9]\,dy = \frac{1}{18}\left[\frac{1}{10}(y+2)^{10}\Big|_0^2 - \frac{1}{10}y^{10}\Big|_{y=0}^{y=2}\right]$

$= \frac{1}{180}[4^{10}-2^{10}-2^{10}]$ ※ or $\frac{2^{18}-2^9}{45}$ ※

#19. $\int_0^{\ln 2}\underbrace{\int_0^{\ln 5} e^{2x-y}\,dx}_{2\cdot\frac{1}{2}}\,dy = \int_0^{\ln 2}\left(\frac{1}{2}e^{2x-y}\Big|_{x=0}^{x=\ln 5}\right)dy$

$e^{2x-y} = e^{2x}\cdot e^{-y}$

$= \int_0^{\ln 2}\left(\frac{1}{2}e^{-y}\cdot 25 - \frac{1}{2}e^{-y}\right)dy$ $\quad e^{2\ln 5} = e^{\ln 5^2} = 25$

$= \frac{-25}{2}e^{-y}\Big|_{y=0}^{y=\ln 2} + \frac{1}{2}e^{-y}\Big|_{y=0}^{y=\ln 2}$

$= \frac{-25}{2}\left[\frac{1}{2}-1\right] + \frac{1}{2}\left[\frac{1}{2}-1\right] = \underline{6}$ ※

〈法二〉

$\int_0^{\ln 2} e^{-y}\,dy \cdot \int_0^{\ln 5} e^{2x}\,dx = -e^{-y}\Big|_0^{\ln 2}\cdot\frac{1}{2}e^{2x}\Big|_0^{\ln 5} = 6$

21~26. Calculate the double integral

#21. $\iint_R \frac{xy^2}{x^2+1}\,dA$, $R=[0,1]\times[-3,3]$

$= \int_0^1 \frac{x}{x^2+1}\,dx \cdot \underbrace{\int_{-3}^3 y^2\,dy}_{\text{有对称性可用}} = \frac{1}{2}\int_0^1 (x^2+1)^{-1}\,d(x^2+1)\cdot 2\cdot\frac{1}{3}y^3\Big|_{y=0}^{y=3}$

$= \frac{1}{2}\ln(x^2+1)\Big|_{x=0}^{x=1}\cdot\frac{2}{3}3^3 = \underline{9\ln 2}$ ※

#23 $\iint_R x\sin(x+y)\,dA$, $R=[0,\frac{\pi}{6}]\times[0,\frac{\pi}{3}]$

先对y积分

$= \int_0^{\frac{\pi}{6}} x\int_0^{\frac{\pi}{3}} \sin(x+y)\,\underset{d(x+y)}{dy}\,dx$

$= \int_0^{\frac{\pi}{6}} x\Big[\underbrace{-\cos(x+y)\Big|_{y=0}^{y=\frac{\pi}{3}}}_{\cos x-\cos(x+\frac{\pi}{3})}\Big]dx = \int_0^{\frac{\pi}{6}} x\cos x\,dx - \int_0^{\frac{\pi}{6}} x\cos(x+\frac{\pi}{3})\,dx$

by integrating by parts 求之

做一下 $\int_0^{\frac{\pi}{6}} x\cos(x+\frac{\pi}{3})\,dx = \int_0^{\frac{\pi}{6}} x\cos(x+\frac{\pi}{3})\,d(x+\frac{\pi}{3}) = \int_0^{\frac{\pi}{6}} x\,d\sin(x+\frac{\pi}{3})$

$= x\sin(x+\frac{\pi}{3})\Big|_{x=0}^{x=\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}}\sin(x+\frac{\pi}{3})\,dx = \frac{\pi}{6}\sin\frac{\pi}{2} + \cos(x+\frac{\pi}{3})\Big|^{x=\frac{\pi}{6}}$

同理 $\int_0^{\frac{\pi}{6}} x\cos x\,dx = \int_0^{\frac{\pi}{6}} x\,d\sin x = x\sin x\Big|_0^{\frac{\pi}{6}} + \cos x\Big|_0^{\frac{\pi}{6}}$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

Thus $\iint_R x\sin(x+y)\,dA = \frac{\sqrt{3}-1}{2} - \frac{\pi}{12}$ #

#25 $\iint_R \underbrace{xye^{x^2y}}_{\text{先对}x\text{积}}\,dA$, $A=[0,1]\times[0,2]$

$= \int_0^2 \int_0^1 \underbrace{xye^{x^2y}\,dx}_{\frac{1}{2}d\,yx^2}\,dy = \int_0^2 \frac{1}{2}e^{x^2y}\Big|_{x=0}^{x=1}\,dy$

$= \frac{1}{2}\int_0^2 (e^y - 1)\,dy$

$= \frac{1}{2}\left[e^y - y\right]\Big|_{y=0}^{y=2} = \frac{1}{2}(e^2-3)$ #

#29. Find the volume of the solid lying under the elliptic paraboloid $\frac{x^2}{4}+\frac{y^2}{9}+z=1$ and above the rectangle $R=[-1,1]\times[-2,2]$

sol: 屋顶 $z = 1-\frac{x^2}{4}-\frac{y^2}{9}$

地基 R

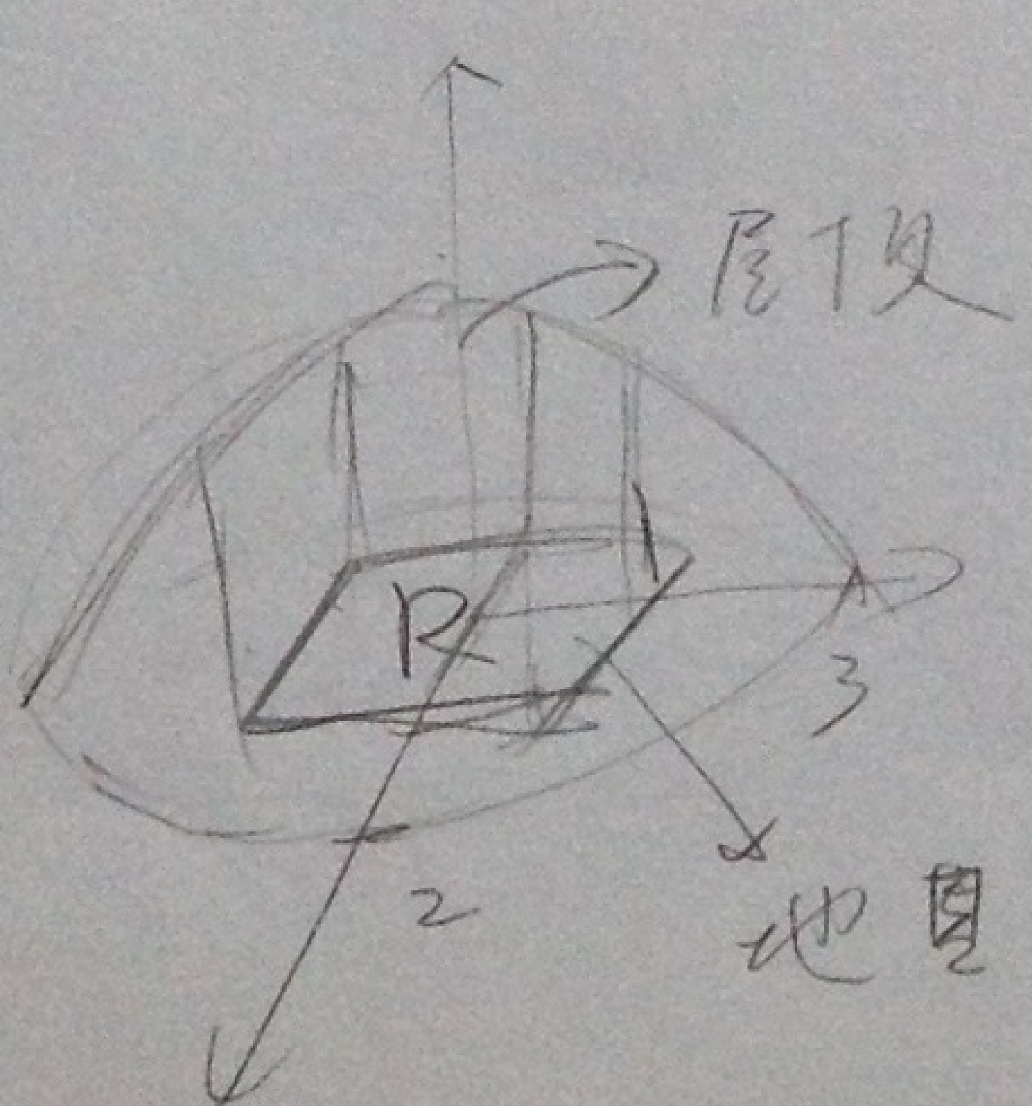
$V = \iint_R \left(1-\frac{x^2}{4}-\frac{y^2}{9}\right)dA$

$= 4\int_0^2 \underbrace{\int_0^1 \left(1-\frac{x^2}{4}-\frac{y^2}{9}\right)dx}\,dy$

使用对称性

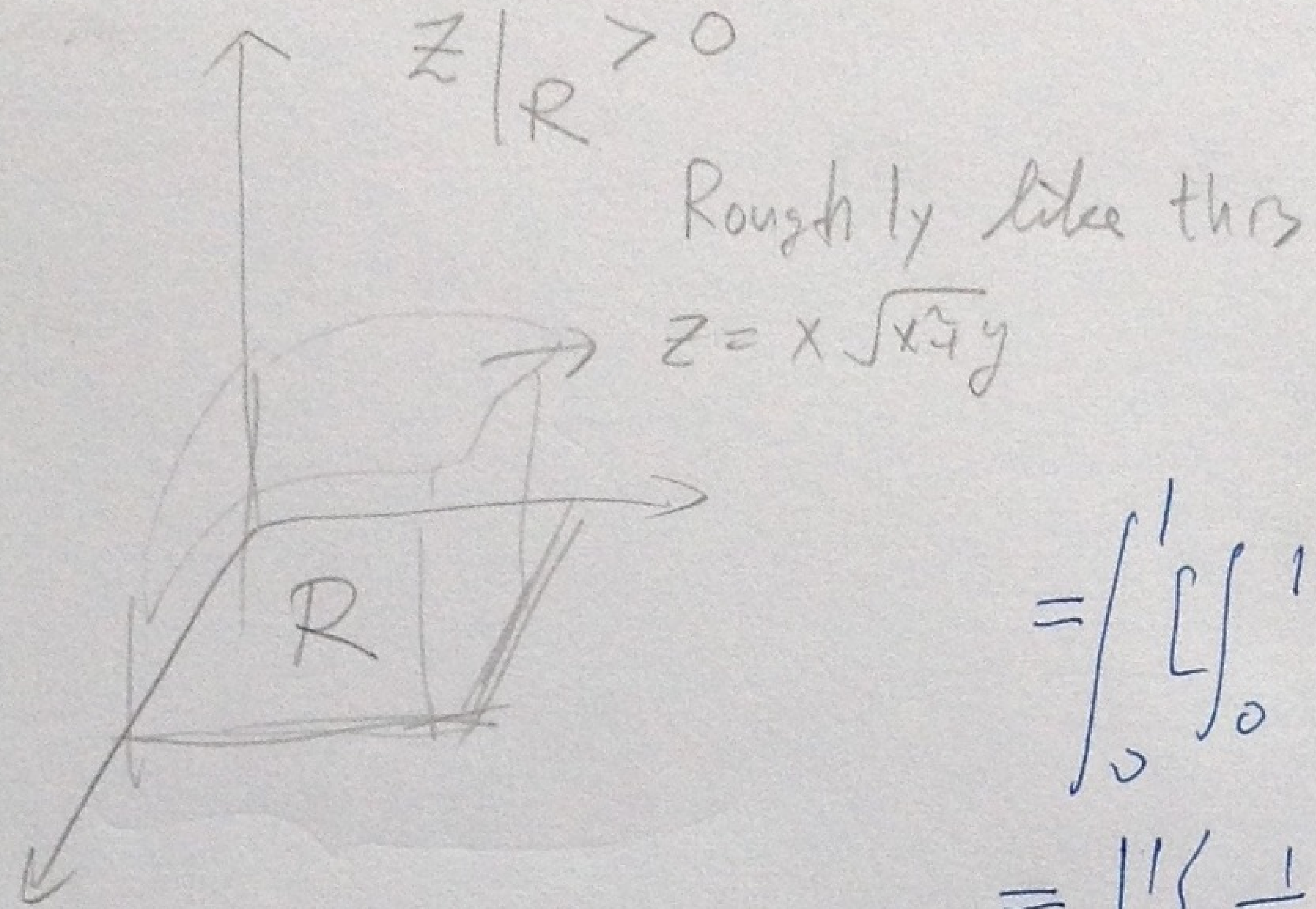
$= 4\int_0^2 \underbrace{\left(x-\frac{1}{12}x^3-\frac{1}{9}y^2x\Big|_{x=0}^{x=1}\right)}_{\frac{11}{12}-\frac{1}{9}y^2}dy$

$= \frac{166}{27}$ #



§4.1

#31. Find the volume of the solid bdd by the surface $z = x\sqrt{x^2+y}$ and the planes $x=0$, $x=1$, $y=0$, $y=1$, and $z=0$



$$V = \iint_R x\sqrt{x^2+y}\, dA$$

$$R = [0,1]\times[0,1]$$

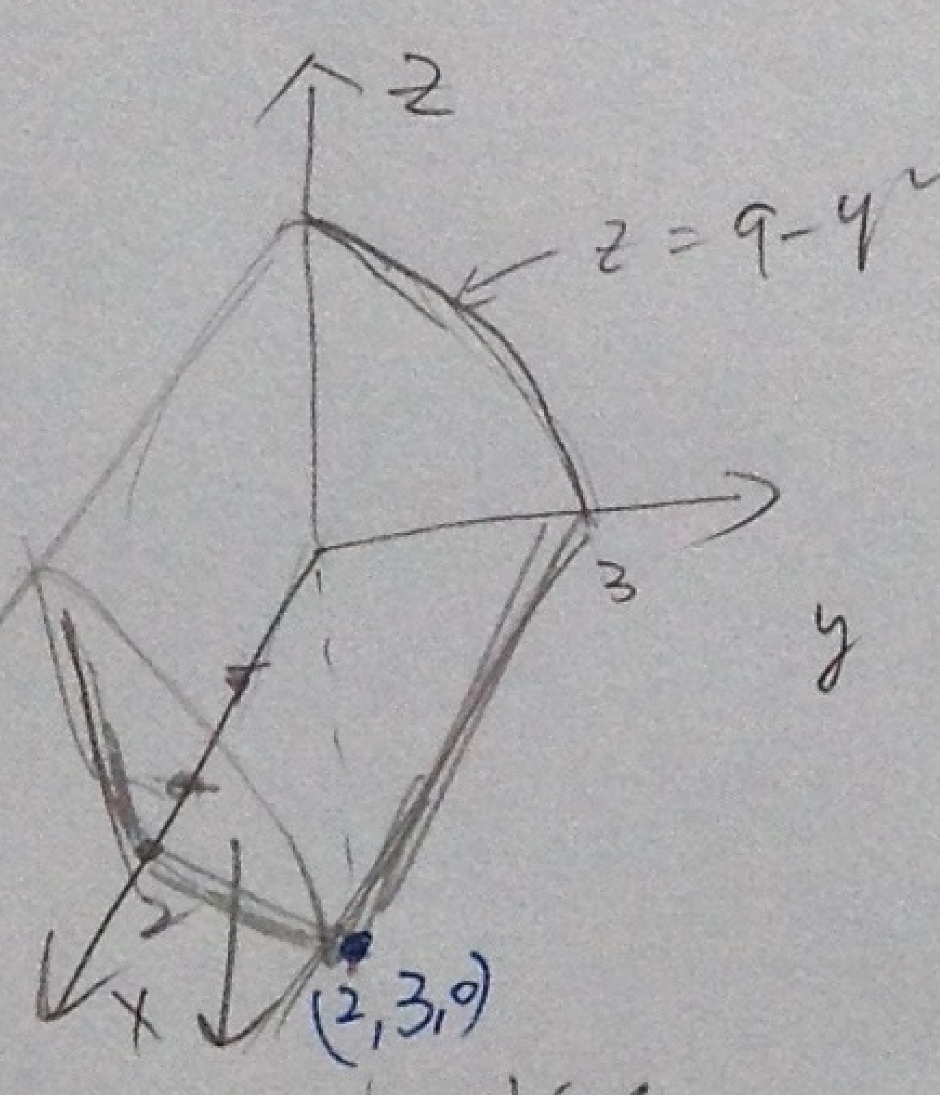
$$= \int_0^1 \left[\int_0^1 (x^2+y)^{\frac{1}{2}}\, dx^2 \cdot \frac{1}{2} \right] dy$$

$$= \int_0^1 \left[\frac{1}{3} (x^2+y)^{\frac{3}{2}} \Big|_{x=0}^{x=1} \right] dy$$

$$= \frac{1}{3} \int_0^1 \left[(y+1)^{\frac{3}{2}} - y^{\frac{3}{2}} \right] dy = \frac{2}{15} \left[(y+1)^{\frac{5}{2}} \Big|_{y=0}^{y=1} - y^{\frac{5}{2}} \Big|_0^1 \right]$$

$$= \frac{4}{15}(2\sqrt{2}-1)$$

#33. Find the volume of the solid in the <u>first octant</u> bdd by the cylinder $z = 9 - y^2$ and the plane $x = 2$.



屋顶 $z = 9 - y^2$

地基 $R = [0,2]\times[0,3]$

$$V = \iint_R (9-y^2)\, dA$$

$$= \int_0^3 \int_0^2 (9-y^2)\, dx\, dy$$

$$= \int_0^3 (9-y^2)\, dy \cdot 2 = 2\left[9y - \frac{1}{3}y^3\right]_{y=0}^{y=3}$$

$$= 2\times 18 = \underline{36}$$